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MAGNETIZED BIANCHI TYPE-III STRING COSMOLOGICAL MODEL FOR ANTI-STIFF FLUID IN GENERAL RELATIVITY

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Abstract

In this paper, we have explored magnetized Bianchi type-III string cosmological model for anti-stiff fluid in general relativity. Anti-stiff fluid models create more interest in the study because these models are free from initial singularity. We assume that F_{12} is the only nonzero component of electromagnetic field tensor F_{ij}. For the complete determination of the model, we assume the expansion in the model is proportional to the shear. The general solution of the Einstein's field equations for the cosmological model have been obtained under the assumption of anti-stiff fluid i.e. $p + \rho = 0$, where ρ and p are the rest energy density and the pressure of the fluid, respectively. The physical and geometrical consequences of the model in the presence of magnetic field are discussed.

Keywords: Bianchi type-III, Anti-stiff fluid, Magnetic field, General Relativity.

1. Introduction

It is still outdare problem for us to know exact physical condition at early phase of formation of the universe. In present days investigations on cosmological models have become dominant and crucial tool in the study of universe and in special case some cosmological model with magnetic field in general relativity and Bianchi type models are more useful and attractive in the study of the evolution during phase transition. One of the simplest anisotropic universe models which plays a crucial role in the understanding of requirement properties of the universe is Bianchi type-III.

String theory purports to be an all-encompassing theory of the universe. The premise of string theory is that, at the fundamental level, matter does not consist of point-particles but rather of tiny loops of string. String theory gives rise to a host of other ingredients, most strikingly extra spatial dimensions of the universe beyond the three that we have observed. Moreover, string theory is very much a work in progress and certain aspects of the theory are far from understood. The general relativistic treatment of strings was initiated by Letelier [7, 8] and Stachel [14]. Letelier [7] investigated the solution of Einstein's field equation for cloud of strings with spherical, plane and cylindrical symmetry.

Magnetic field plays a vital role in description of the energy distribution in the universe. Strong magnetic fields can be created due to adiabatic compression in cluster galaxies. The magnetic field has the significance role in the dynamics of the field lines. The importance of the magnetic field for various astrophysical phenomenon has been studied in many papers.

Lorentz [9] has presented tilted electromagnetic Bianchi type III cosmological solution. Tikekar and Patel [15,16] obtained some exact solutions of massive string of Bianchi type –III space time in the presence and absence of magnetic field. The string cosmological models with magnetic field are investigated by Chakraborty [5], Patel and Maharaj [10], Singh and Singh [13], Bali and Upadhaya [3].

Bali and Jain [2] have studied Bianchi type –III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Bali and Pareek [4] have presented massive string of Bianchi type III cosmological model for perfect fluid distribution in the presence of magnetic field. Upadhaya and Dave [20] have investigated some magnetized Bianchi type –III Massive string Cosmological models in general relativity. Amirhaschi H. Zainuddin [1] studied Magnetized Bianchi type III massive string cosmological model in general relativity.

Pradhan et al. [12] has presented Massive string cosmology in Bianchi type III space-time with electromagnetic field. Pradhan et al. [11] derived Generation of bulk viscous fluid massive string cosmological models with electromagnetic field in Bianchi type-VI_0 space-time.

Deo et al. [6] have investigated Bianchi type-III cosmological model electromagnetic field with cosmic string in general theory of relativity.

Tyagi and Chhajed [17] investigated homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electro-magnetic field. Tyagi et al. [18] investigated Magnetized LRS Bianchi Type-I massive string cosmological model for perfect fluid distribution with cosmological term Λ. Tyagi et al. [19] investigated Magnetized Bianchi Type-VI_0 cosmological model for barotropic fluid distribution with variable magnetic permeability and dark energy.

Motivated by the above research work, in this paper, we have investigated magnetized Bianchi type-III string cosmological model for anti-stiff fluid in general relativity. The physical and geometrical consequences of the model in the presence of magnetic field are discussed.

2. The Metric and Field Equations

We consider spatially homogeneous anisotropic Bianchi type-III space-time as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2dz^2 \tag{1}$$

where A, B and C are function of cosmic time t and m is non-zero constant.

Einstein's field equation is given by

$$R_i^j - \frac{R}{2} g_i^j = -8\pi T_i^j \tag{2}$$

where T_i^j is the energy momentum tensor for a cloud of strings given by Letelier (1979, 1983) as

$$T_i^j = (p + \rho) v_i v^j + p g_i^j - \lambda x_i x^j + E_i^j \tag{3}$$

with

$$v_i v^i = -x_i x^i = -1 \text{ and} v^i x_i = 0 \tag{4}$$

where λ is the string tension density, ρ is the matter density, p is the thermodynamical pressure, v^i is the four-velocity vector and x^i the direction of string. The particle density associated with the configuration is given by

$$\rho_p = p - \lambda \tag{5}$$

E_i^j is the electromagnetic field given by

$$E_i^j = \frac{1}{4\pi}\left[g^{rs} F_{ir} F_s^j - \frac{1}{4} F_{rs} F^{rs} g_i^j\right] \tag{6}$$

We assume that the current is flowing along z-axis so magnetic field is in the xy-plane. Thus F_{12} is the only non-vanishing component of $F_{ij.}$

The Maxwell's equation

$$\frac{\partial}{\partial x^j}\left(F^{12}\sqrt{-g}\right) = 0 \tag{7}$$

leads to

$$F_{12} = Ke^{-mx} \tag{8}$$

where K is constant.

Now the non-vanishing components of E_i^j corresponding to the line-element (1) are given as follow:

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{K^2}{8\pi A^2 B^2} \tag{9}$$

In the above v^i is the flow vector satisfying

$$g_{ij} v^i v^j = -1 \tag{10}$$

and direction of string is along z-axis so that $x_1 = 0$, $x_2 = 0$, $x_3 \neq 0$, $x_4 = 0$.

We assume the coordinates to be comoving so that

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = 1 \tag{11}$$

The Einstein's field equation (2) for the metric (1) leads to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-8\pi p-\frac{K^2}{A^2B^2} \quad (12)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=-8\pi p-\frac{K^2}{A^2B^2} \quad (13)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{m^2}{A^2}=-8\pi(p-\lambda)+\frac{K^2}{A^2B^2} \quad (14)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{m^2}{A^2}=8\pi\rho+\frac{K^2}{A^2B^2} \quad (15)$$

and

$$m\left[\frac{A_4}{A}-\frac{B_4}{B}\right]=0 \quad (16)$$

Since $m \neq 0$, Equation (16) leads to

$$A=\mu B \quad (17)$$

where μ is an integrating constant. We consider A=B, taking μ=1 without loss of generality then the field equations (12) to (15) reduces to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-8\pi p-\frac{K^2}{B^4} \quad (18)$$

$$2\frac{B_{44}}{B}+\frac{{B_4}^2}{B^2}-\frac{m^2}{B^2}=-8\pi(p-\lambda)+\frac{K^2}{B^4} \quad (19)$$

$$\frac{{B_4}^2}{B^2}+2\frac{B_4C_4}{BC}-\frac{m^2}{B^2}=8\pi\rho+\frac{K^2}{B^4} \quad (20)$$

Now, we describe some criterions for Bianchi type-III model which are crucial in cosmological observations. The spatial volume V is defined as

$$V=B^2C \quad (21)$$

The expansion (θ) and shear scalar (σ) are given by

$$\theta=2\frac{B_4}{B}+\frac{C_4}{C} \quad (22)$$

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{B_4}{B}-\frac{C_4}{C}\right) \quad (23)$$

The generalized mean Hubble parameter H is defined by

$$H=\frac{\theta}{3}=\frac{1}{3}\left(2\frac{B_4}{B}+\frac{C_4}{C}\right) \quad (24)$$

A physical quantity deceleration parameter q defined by

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right) \quad (25)$$

3. Solution of the Field Equations

Here we have three non-linear differential equations (18) - (20) in five unknowns B, C, p, λ and ρ. In order to obtain consistent solutions, we need two extra conditions. To get the deterministic model, we assume that the expansion (θ) is proportional to the shear (σ).

This leads to condition

$$C = B^n \tag{26}$$

And we assume anti-stiff fluid i.e.

$$p + \rho = 0 \tag{27}$$

from equations (18) and (20), after using equation (27), we get

$$\frac{B_4{}^2}{B^2} + \frac{B_4C_4}{BC} - \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{m^2}{B^2} = 2\frac{K^2}{B^4} \tag{28}$$

Combining (21) and (26), we get

$$B = V^{1/n+2},\ C = V^{n/n+2} \tag{29}$$

Substituting (29) into (28), we obtain

$$-\frac{(n+1)}{(n+2)}\frac{V_{44}}{V} + \frac{4n+2}{(n+2)^2}\frac{V_4{}^2}{V^2} = \frac{m^2}{V^{2/(n+2)}} + \frac{2K^2}{V^{4/(n+2)}} \tag{30}$$

Equation (30) leads to

$$V_{44} - \frac{(4n+2)}{(n+1)(n+2)}\frac{V_4{}^2}{V} = -\frac{(n+2)}{(n+1)}\frac{m^2}{V^{-n/(n+2)}} - \frac{2(n+2)}{(n+1)}\frac{K^2}{V^{-(\frac{n-2}{n+2})}} \tag{31}$$

Let us considerV_4=f(V) and

$V_{44} = ff^1$, $f^1 = \frac{df}{dV}$ in equation (31)

We get,

$$ff^1 - \frac{(4n+2)}{(n+1)(n+2)}\frac{f^2}{V} = -\frac{(n+2)}{(n+1)}\frac{m^2}{V^{-n/(n+2)}} - \frac{2(n+2)}{(n+1)}\frac{K^2}{V^{-(\frac{n-2}{n+2})}} \tag{32}$$

Equation (32) can be written as

$$\frac{df^2}{dV} - \frac{2(4n+2)}{(n+1)(n+2)}\frac{f^2}{V} = -\frac{2(n+2)}{(n+1)}\frac{m^2}{V^{-n/(n+2)}} - \frac{4(n+2)}{(n+1)}\frac{K^2}{V^{-(\frac{n-2}{n+2})}} \tag{33}$$

On integrating equation (33), we get

$$f^2 = V_4{}^2 = -\frac{m^2(n+2)^2}{n^2-2n-1}V^{\frac{2(n+1)}{(n+2)}} - \frac{2K^2(n+2)^2}{n^2-3n-2}V^{\frac{2n}{(n+2)}} + LV^{\frac{2(4n+2)}{(n+1)(n+2)}} \tag{34}$$

where L is the integrating constant.

Equation (34) leads to

$$\int \frac{dV}{\sqrt{LV^{\frac{2(4n+2)}{(n+1)(n+2)}} - \frac{m^2(n+2)^2}{n^2-2n-1} V^{\frac{2(n+1)}{(n+2)}} - \frac{2K^2(n+2)^2}{n^2-3n-2} V^{\frac{2n}{(n+2)}}}} = \int dt + M = t + M \tag{35}$$

where M is the integrating constant.

Appropriate transformation of coordinates

V =T, x =X, y =Y, z = Z

The metric (1) becomes,

$$ds^2 = -\frac{dT^2}{LT^{\frac{2(4n+2)}{(n+1)(n+2)}} - \frac{m^2(n+2)^2}{n^2-2n-1} T^{\frac{2(n+1)}{(n+2)}} - \frac{2K^2(n+2)^2}{n^2-3n-2} T^{\frac{2n}{(n+2)}}} + T^{2/(n+2)}(dX^2 + e^{-2mX}dY^2) + T^{2n/(n+2)}dZ^2 \tag{36}$$

4. Special Models:

I. For $n = 2$ and $L = 0$ in the presence of magnetic field

When, we put $n = 2$ and $L = 0$ in equation (34) it leads to

$$f^2 = {V_4}^2 = 8V(2m^2V^{\frac{1}{2}} + K^2) \tag{37}$$

Equation (37) can be rewrite as

$$\int \frac{dV}{\sqrt{8V(2m^2V^{\frac{1}{2}} + K^2)}} = \int dt \tag{38}$$

On integrating equation (38), we get

$$V = \frac{1}{4m^4}(T^2 - K^2)^2 \tag{39}$$

where $\sqrt{2}m^2(t + S) = T$ (40)

and S is integrating constant.

Therefore, metric (1) reduces to the form

$$ds^2 = -\frac{dT^2}{2m^4} + \frac{1}{2m^2}(T^2 - K^2)(dX^2 + e^{-2mX}dY^2) + \frac{1}{4m^4}(T^2 - K^2)^2dZ^2 \tag{41}$$

II. For $n = 2$ and $L = 0$ in the absence of Magnetic field

When, we put $n = 2$, $L = 0$ and $K = 0$ in equation (34) it leads to

$$f^2 = {V_4}^2 = 16m^2V^{\frac{3}{2}} \tag{42}$$

Equation (42) can be rewrite as

$$\int \frac{dV}{4V^{\frac{3}{4}}} = m \int dt \tag{43}$$

On integrating equation (43), we get

$$V = T^4 \tag{44}$$

where $mt + C_1 = T$ (45)

and C_1 is integrating constant.

Therefore, metric (1) reduces to the form

$$ds^2 = -\frac{dT^2}{m^2} + T^2(dX^2 + e^{-2mX}dY^2) + T^4 dZ^2 \tag{46}$$

5. The Geometrical and Physical significance of models:

For the model (36), the energy density (ρ), isotropic pressure (p), string tension density (λ), particle density (ρ_p) attached to the string, the expansion (θ), shear (σ), Hubble parameter (H) and deceleration parameter (q) are given by

$$8\pi\rho = \frac{(2n+1)}{(n+2)^2}\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2n^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{K^2(n^2+n)}{(n^2-3n-2)\,T^{\frac{4}{(n+2)}}} \tag{47}$$

$$8\pi p = -\left(\frac{(2n+1)}{(n+2)^2}\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2n^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{K^2(n^2+n)}{(n^2-3n-2)\,T^{\frac{4}{(n+2)}}}\right) \tag{48}$$

$$8\pi\lambda = \frac{2(-2n^2+n+1)}{(n+1)(n+2)^2}\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} + \frac{2m^2n}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} + \frac{4K^2(n+1)}{(n^2-3n-2)T^{\frac{4}{(n+2)}}} \tag{49}$$

$$8\pi\rho_p = \frac{(6n^2+n-1)}{(n+1)(n+2)^2}\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2n(n+2)}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{K^2(n^2+5n+4)}{(n^2-3n-2)T^{\frac{4}{(n+2)}}} \tag{50}$$

$$\theta = \sqrt{\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2(n+2)^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{2K^2(n+2)^2}{(n^2-3n-2)T^{\frac{4}{(n+2)}}}} \tag{51}$$

$$\sigma = \frac{(1-n)}{\sqrt{3}(n+2)}\sqrt{\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2(n+2)^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{2K^2(n+2)^2}{(n^2-3n-2)T^{\frac{4}{(n+2)}}}} \tag{52}$$

From (51) and (52), we get

$$\frac{\sigma}{\theta} = \frac{(1-n)}{\sqrt{3}(n+2)} = \text{Constant},\ (n \neq -2) \tag{53}$$

$$H = \frac{1}{3}\left(\sqrt{\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2(n+2)^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{2K^2(n+2)^2}{(n^2-3n-2)T^{\frac{4}{(n+2)}}}}\right) \tag{54}$$

$$q = -1 - \frac{3}{L}\left\{\frac{\frac{2n(1-n)L}{(n+1)(n+2)T^{\frac{2n(n-1)}{(n+1)(n+2)}}} + \frac{2m^2(n+2)}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} + \frac{4K^2(n+2)}{(n^2-3n-2)T^{\frac{4}{(n+2)}}}}{\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2(n+2)^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{2K^2(n+2)^2}{(n^2-3n-2)T^{\frac{4}{(n+2)}}}}\right\} \tag{55}$$

Energy condition $\rho \geq 0$ leads to

$$\left(\frac{(2n+1)}{(n+2)^2}\frac{L}{T^{\frac{2n(n-1)}{(n+1)(n+2)}}} - \frac{m^2n^2}{(n^2-2n-1)T^{\frac{2}{(n+2)}}} - \frac{K^2(n^2+n)}{(n^2-3n-2)\,T^{\frac{4}{(n+2)}}}\right) \geq 0 \tag{56}$$

The magnitude of rotation

$$\omega = 0 \tag{57}$$

Now for the special model (41), the energy density (ρ), isotropic pressure (p), string tension density (λ), particle density (ρ_p) attached to the string, the expansion (θ), shear (σ), Hubble parameter (H) and deceleration parameter (q) are given by

$$8\pi\rho = \frac{2m^4(4T^2-K^2)}{(T^2-K^2)^2} \tag{58}$$

$$8\pi p = -\left(\frac{2m^4(4T^2-K^2)}{(T^2-K^2)^2}\right) \tag{59}$$

$$8\pi\lambda = -\frac{4m^4(2T^2+K^2)}{(T^2-K^2)^2} \tag{60}$$

$$8\pi\rho_p = \frac{2m^4(8T^2+K^2)}{(T^2-K^2)^2} \tag{61}$$

$$\theta = \frac{4\sqrt{2}m^2T}{T^2-K^2} \tag{62}$$

$$\sigma = -\frac{\sqrt{2}m^2T}{T^2-K^2} \tag{63}$$

From (62) and (63), we get

$$\frac{\sigma}{\theta} = -\frac{1}{4} = \text{Constant} \tag{64}$$

$$H = \frac{4\sqrt{2}m^2T}{3(T^2-K^2)} \tag{65}$$

$$q = -1 + \frac{3\sqrt{2}m^2(T^2+K^2)}{4T^2} \tag{66}$$

Energy condition $\rho \geq 0$ leads to

$$\frac{2m^4(4T^2-K^2)}{(T^2-K^2)^2} \geq 0 \tag{67}$$

The magnitude of rotation

$$\omega = 0 \tag{68}$$

For the special model (46), the energy density (ρ), isotropic pressure (p), string tension density (λ), particle density (ρ_p) attached to the string, the expansion (θ), shear (σ), Hubble parameter (H) and deceleration parameter (q) are given by

$$8\pi\rho = \frac{4m^2}{T^2} \tag{69}$$

$$8\pi p = -\frac{4m^2}{T^2} \tag{70}$$

$$8\pi\lambda = -\frac{4m^2}{T^2} \tag{71}$$

$$8\pi\rho_p = \frac{8m^2}{T^2} \tag{72}$$

$$\theta = \frac{4m}{T} \tag{73}$$

$$\sigma = -\frac{m}{\sqrt{3}T} \tag{74}$$

From (73) and (74), we get

$$\frac{\sigma}{\theta} = -\frac{4}{\sqrt{3}} = \text{Constant} \tag{75}$$

$$H = \frac{4m}{3T} \tag{76}$$

$$q = -\frac{1}{4} \tag{77}$$

Energy condition $\rho \geq 0$ leads to

$$\frac{4m^2}{T^2} \geq 0 \tag{78}$$

The magnitude of rotation

$$\omega = 0 \tag{79}$$

6. Conclusion

In this paper, we have explored Magnetized Bianchi type-III string cosmological model for anti-stiff fluid in General Relativity, we get a new exact solution of Einstein's field equations. The model (36) starts expanding with big bang at T = 0 and the expansion of the model decreases as time increases, for $n > -1$.The expansion stops as T $\to\infty$. As T $\to$0, V$\to$0 and

V is the increasing function of T. When $n = -1$, T tends to infinity, q tends to -1. The comparative extent of the shear scalar σ and expansion θ tends to finite value *i.e.* $\frac{\sigma}{\theta} = \frac{(1-n)}{\sqrt{3}(n+2)}$, (n ≠ -2). Since T $\to\infty$, $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of T. However, if $n = 1$ then $\sigma = 0$. Thus the model isotropizes for $n = 1$. We observe that the energy density, pressure, string density, particle density and Hubble parameter are decreasing function of time and tends to zero as T $\to\infty$, $n > -1$. The energy condition $\rho \geq 0$ is satisfied for all values of T. Expansion in the model (41) is decreasing function of cosmic time T , the expansion stop as T $\to\infty$. For large value of T and $m^2 < \frac{4}{3\sqrt{2}}$ value of q lie between -1 to 0 which is good agreement with present astronomical observations.

All physical and Kinematical parameters tends to 0 as T $\rightarrow \infty$. The energy condition $\rho \geq 0$ is satisfied for all values of T. Since T$\rightarrow \infty$, $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of T. All physical parameters decreases more rapidly in the presence of magnetic field.

In the absence of magnetic field, the model (46) starts expanding with big bang at $T = 0$ and the expansion of the model decreases as time increases and the expansion stop as T $\rightarrow \infty$. The value of deceleration parameter obtained is -0.25 which lie between -1 to 0. All parameters tends to 0 as T $\rightarrow \infty$. The energy condition $\rho \geq 0$ is satisfied for all values of T. Since T $\rightarrow \infty$, $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of T.

The models (36) and (46) have point type singularity at $T = 0$. In general, the present model represents shearing, expanding and non-rotating universe.

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